GCE214
Applied Mechanics-Statics

Lecture 05: 04/10/2017

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Class: Wednesday (3–5 pm)
Venue: LT1

Etiquettes and MOP

- Attendance is a requirement.
- There may be class assessments, during or after lecture.
- Computational software will be employed in solving problems
- Conceptual understanding will be tested
- Lively discussions are integral part of the lectures.

Lecture content

Rigid Bodies: Equivalent Systems of Forces

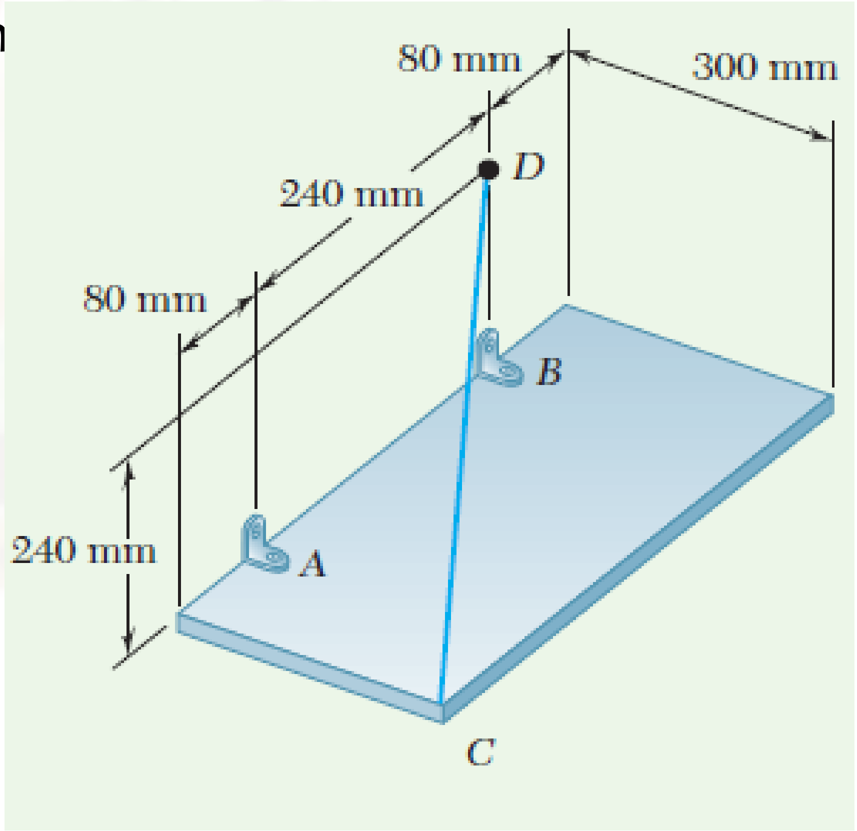
- Moment of a force about a given Axis
- Moment of a couple
- Couple vectors
- Resolution of a given force into a force at *O* and a couple

Recommended textbook

- Vector Mechanics for Engineers: Statics and Dynamics by Beer, Johnston, Mazurek, Cornwell. 10^{th} Edition

EXAMPLES

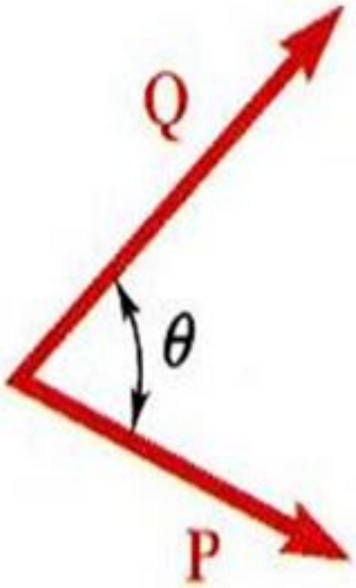
1. A rectangular plate is supported by brackets at *A* and *B* and by a wire *CD*. Knowing that the tension in the wire is 200 N, determine the moment about 80 mm *A* of the force exerted by the wire on poin



SCALAR PRODUCT OF TWO VECTORS

- The scalar product or dot product between two vectors **P** and **Q** is defined as

  $\boldsymbol{P} \bullet \boldsymbol{Q} = PQ \cos\theta$ (scalar result)



- Scalar products:
  - are commutative, $\boldsymbol{P} \bullet \boldsymbol{Q} = \boldsymbol{Q} \bullet \boldsymbol{P}$
  - are distributive, $\boldsymbol{P} \bullet (\boldsymbol{Q}_1 + \boldsymbol{Q}_2) = \boldsymbol{P} \bullet \boldsymbol{Q}_1 + \boldsymbol{P} \bullet \boldsymbol{Q}_2$
  - are not associative, $(\boldsymbol{P} \bullet \boldsymbol{Q}) \bullet \boldsymbol{S} = undefined$

- Scalar products with Cartesian unit components,

$$\boldsymbol{P} \bullet \boldsymbol{Q} = \left(P_x\boldsymbol{i} + P_y\boldsymbol{j} + P_z\boldsymbol{k}\right) \bullet \left(Q_x\boldsymbol{i} + Q_y\boldsymbol{j} + Q_z\boldsymbol{k}\right)$$

$$\boldsymbol{i} \bullet \boldsymbol{i} = 1, \quad \boldsymbol{j} \bullet \boldsymbol{j} = 1, \quad \boldsymbol{k} \bullet \boldsymbol{k} = 1, \quad \boldsymbol{i} \bullet \boldsymbol{j} = 0, \quad \boldsymbol{j} \bullet \boldsymbol{k} = 0, \quad \boldsymbol{k} \bullet \boldsymbol{i} = 0$$

$$\boldsymbol{P} \bullet \boldsymbol{Q} = P_xQ_x + P_yQ_y + P_zQ_z$$

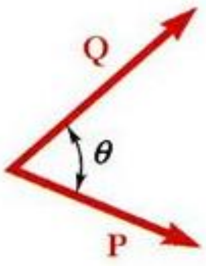
$$\boldsymbol{P} \bullet \boldsymbol{P} = P_x^2 + P_y^2 + P_z^2$$

SCALAR PRODUCT OF TWO VECTORS: APPLICATIONS

- Angle between two vectors:

$$\bar{P} \bullet \bar{Q} = PQ\cos\theta = P_x Q_x + P_y Q_y + P_z Q_z$$

$$\cos\theta = \frac{P_x Q_x + P_y Q_y + P_z Q_z}{PQ}$$

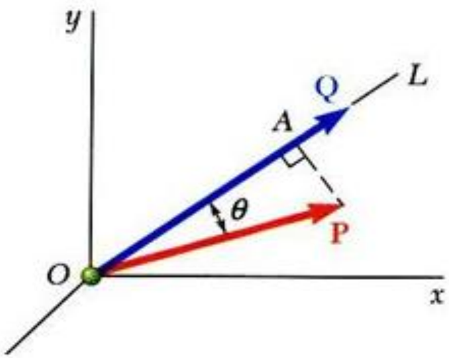


- Projection of a vector on a given axis:

$$P_{OL} = P\cos\theta = \text{projection of } P \text{ along } OL$$

$$\bar{P} \bullet \bar{Q} = PQ\cos\theta$$

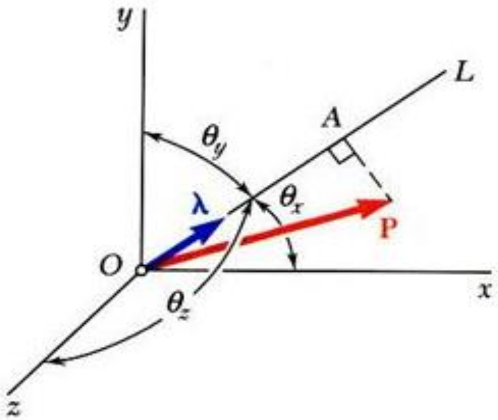
$$\frac{\bar{P} \bullet \bar{Q}}{Q} = P\cos\theta = P_{OL}$$



- For an axis defined by a unit vector:

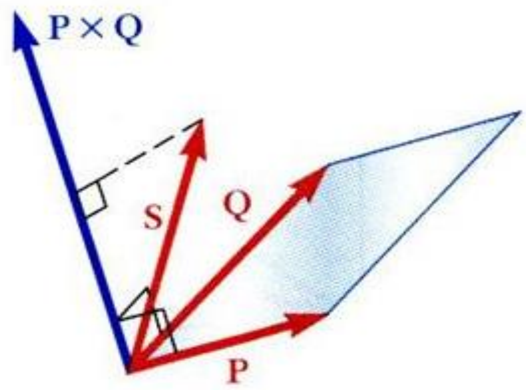
$$P_{OL} = \bar{P} \bullet \bar{\lambda}$$

$$= P_x\cos\theta_x + P_y\cos\theta_y + P_z\cos\theta_z$$



RIGID BODIES: EQUIVALENT SYSTEMS OF FORCES

MIXED TRIPLE PRODUCT OF THREE VECTORS



- Mixed triple product of three vectors,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=\text{scalar result}$$

- The six mixed triple products formed from ***S***, ***P***, and ***Q*** have equal magnitudes but not the same sign,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=\bar{P}\bullet\left(\bar{Q}\times\bar{S}\right)=\bar{Q}\bullet\left(\bar{S}\times\bar{P}\right)$$
$$=-\bar{S}\bullet\left(\bar{Q}\times P\right)=-\bar{P}\bullet\left(\bar{S}\times\bar{Q}\right)=-\bar{Q}\bullet\left(\bar{P}\times\bar{S}\right)$$

- Evaluating the mixed triple product,

$$\bar{S}\bullet\left(\bar{P}\times\bar{Q}\right)=S_x\left(P_yQ_z-P_zQ_y\right)+S_y\left(P_zQ_x-P_xQ_z\right)$$
$$+S_z\left(P_xQ_y-P_yQ_x\right)$$
$$=\begin{vmatrix} S_x & S_y & S_z \\ P_x & P_y & P_z \\ Q_x & Q_y & Q_z \end{vmatrix}$$

MOMENT OF A ABOUT A GIVEN AXIS

- Moment $\boldsymbol{M_O}$ of a force $\boldsymbol{F}$ applied at the point $\boldsymbol{A}$ about a point $\boldsymbol{O}$,

$$\bar{M}_O = \bar{r} \times \bar{F}$$

- Scalar moment M_{OL} about an axis $\boldsymbol{OL}$ is the projection of the moment vector $\boldsymbol{M_O}$ onto the axis,

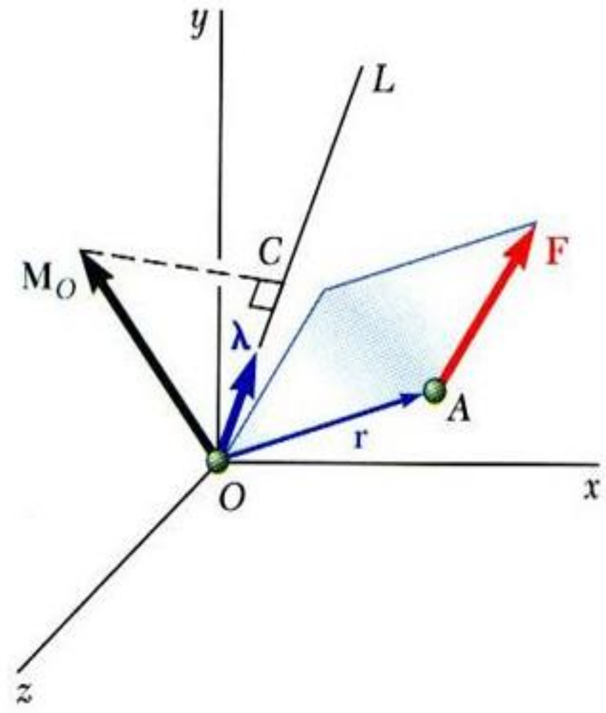
$$M_{OL} = \bar{\lambda} \bullet \bar{M}_O = \bar{\lambda} \bullet \left(\bar{r} \times \bar{F}\right)$$

- Moments of $\boldsymbol{F}$ about the coordinate axes,

$$M_x = yF_z - zF_y$$

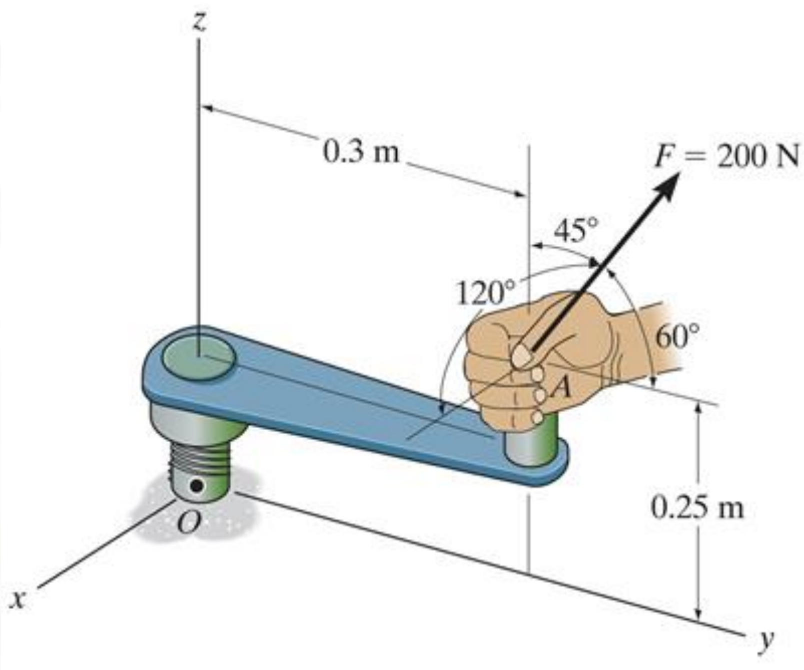
$$M_y = zF_x - xF_z$$

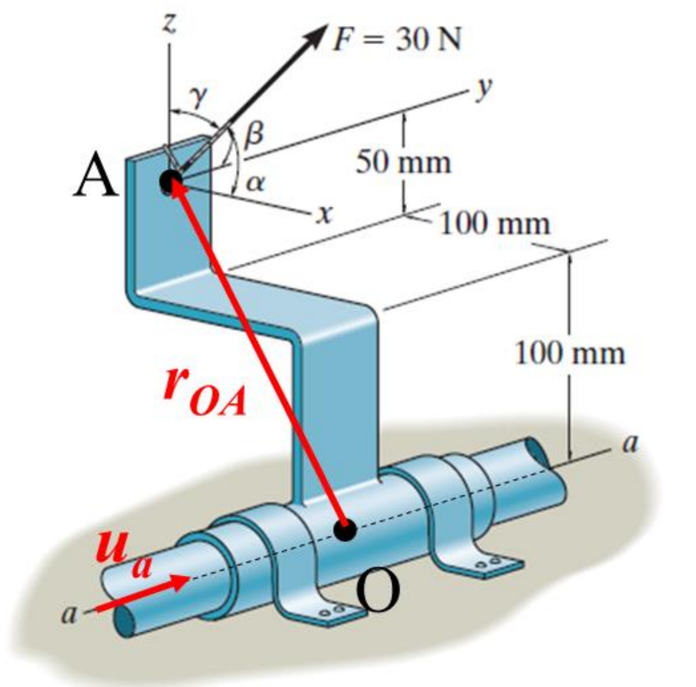
$$M_z = xF_y - yF_x$$



EXAMPLES

2. Find the magnitude of the moment of the force about the x-axis for a force 200 N acting as shown in the figure.

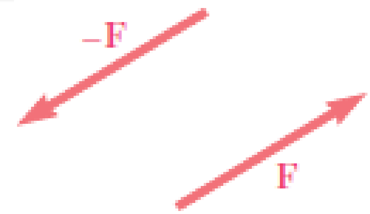




3. The force F acts on a bracket as shown in the figure. Take $\alpha = 60°$, $\beta = 60°$, $\gamma = 45°$. Find the magnitude of the moment about a-a axis.

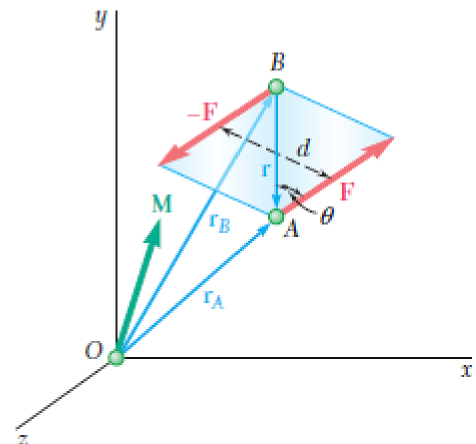
MOMENT OF A COUPLE

- A couples consist of two forces of equal magnitude, parallel lines of action and opposite sense.



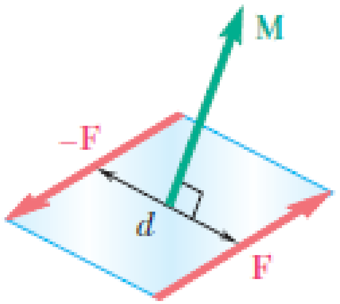
- The sum of the moments of **F** and **–F** about *O* is represented by the vector **M** as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$



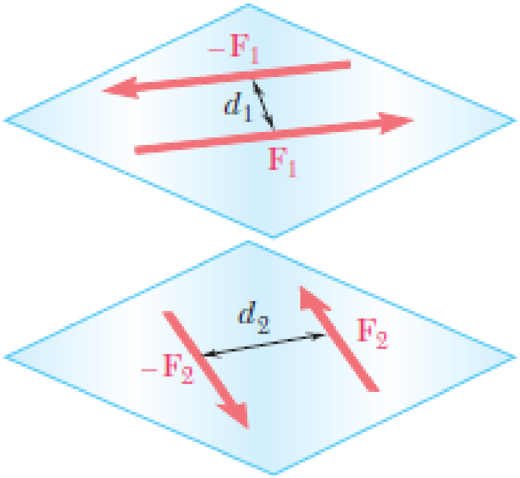
- The vector **M** is called the *moment of the couple*. It is perpendicular to the plane containing the two forces and its magnitude is given as

$$M = rF \sin\theta = Fd$$



EQUIVALENT COUPLES

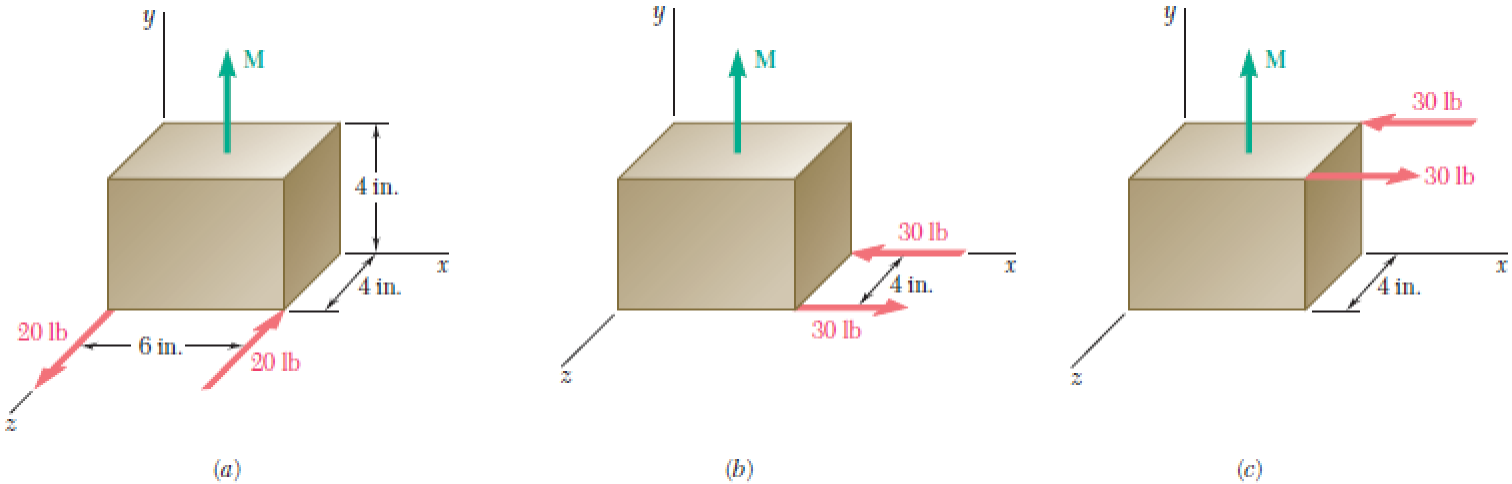
- Consider two couples as shown in the Figure – one with $\boldsymbol{F_1}$ and $-\boldsymbol{F_1}$ and the other $\boldsymbol{F_2}$ and $-\boldsymbol{F_2}$.



- It may be concluded that both have equal moments

$$F_1 d_1 = F_2 d_2$$

provided the two couples are lie in parallel planes (or in the same plane) and have the same sense (i.e. clockwise or counterclockwise)



- Three equivalent couples producing same moment are shown above

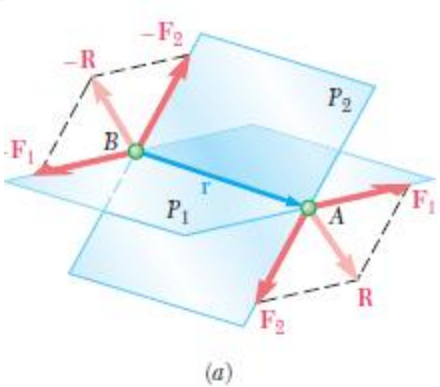
COUPLES AND FORCE COUPLE SYSTEMS

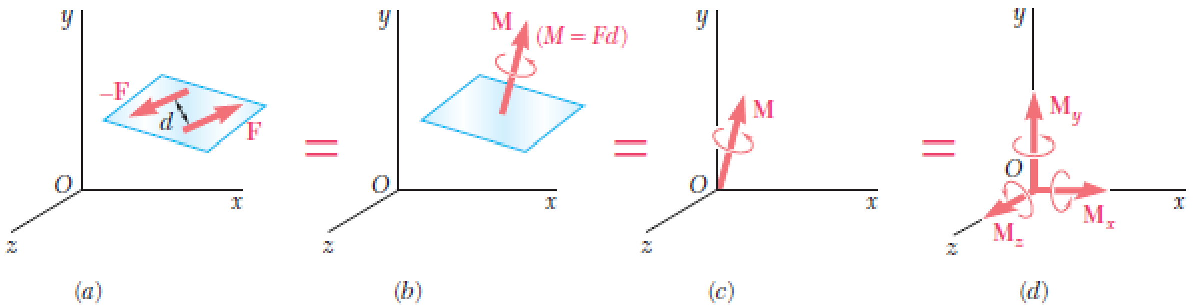
ADDITION OF COUPLES AND COUPLE VECTORS

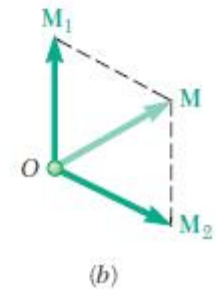
- Addition of two couples, each acting in one of two intersecting planes, to form a new couple is possible.
- The moment of the resultant couple is the vector sum of the moments of the component couples.

$$\mathbf{M} = \mathbf{M}_1 + \mathbf{M}_2$$

$$\mathbf{M} = \Sigma \mathbf{M} = \Sigma(\mathbf{r} \times \mathbf{F})$$



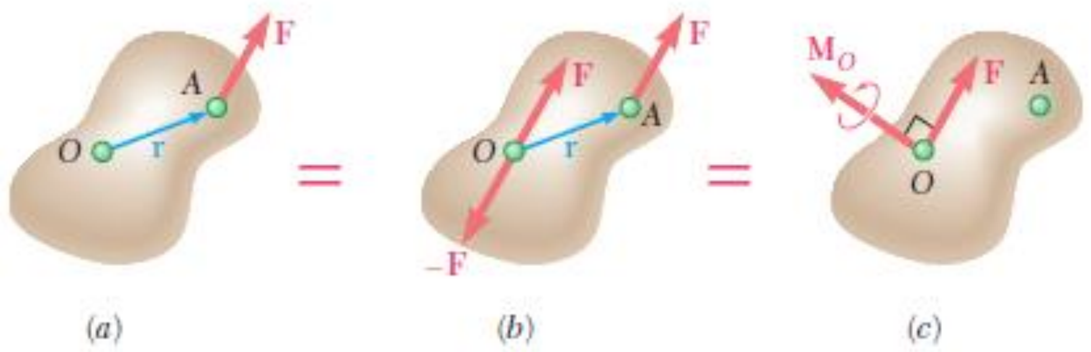




- (*a*) A couple formed by two forces can be represented by (*b*) a couple vector, oriented perpendicular to the plane of the couple. (*c*) The couple vector is a free vector and can be moved to other points of application, such as the origin. (*d*) A couple vector can be resolved into components along the coordinate axes.

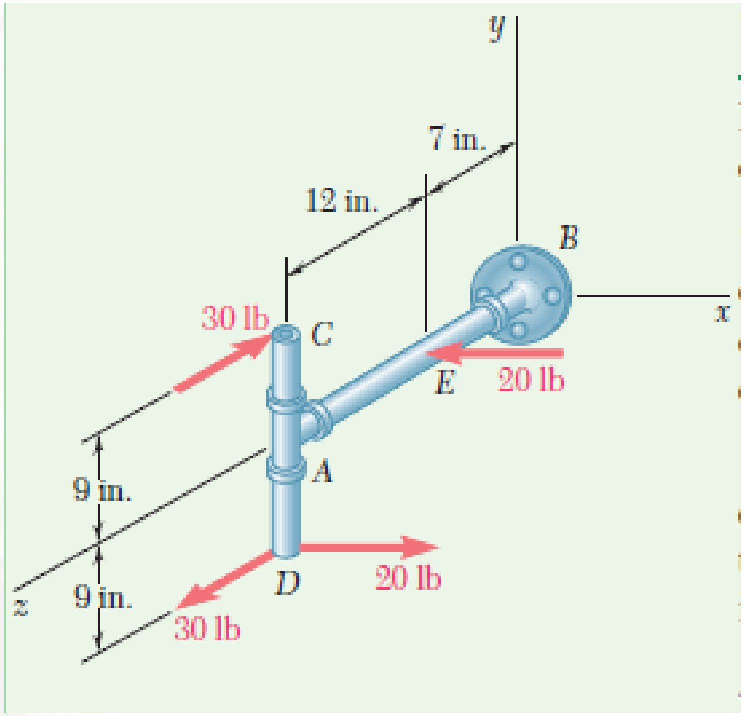
RESOLUTION OF A GIVEN FORCE INTO A FORCE AT O AND A COUPLE

- Replacing a force with a force and a couple. (*a*) Initial force **F** acting at point *A*; (*b*) attaching equal and opposite forces at *O*; (*c*) force **F** acting at point *O* and a couple.

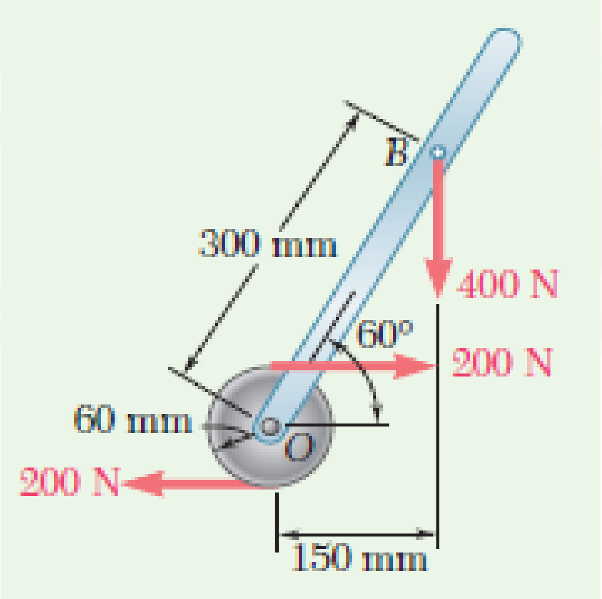


EXAMPLES

4. Determine the components of the single couple equivalent to the two couples shown.

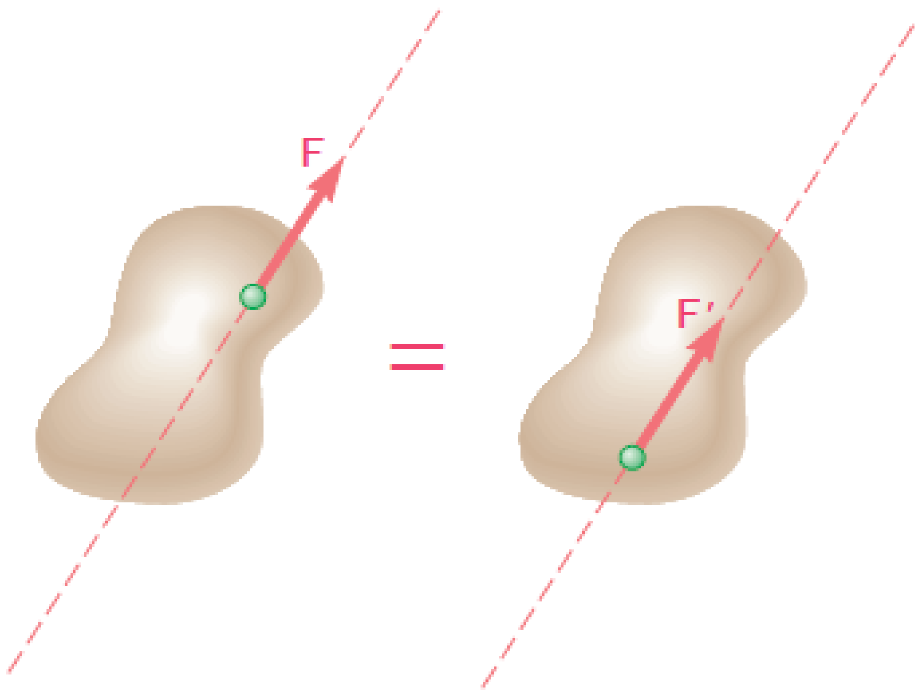


5. Replace the couple and force shown by an equivalent single force applied to the lever. Determine the distance from the shaft to the point of application of this equivalent force.

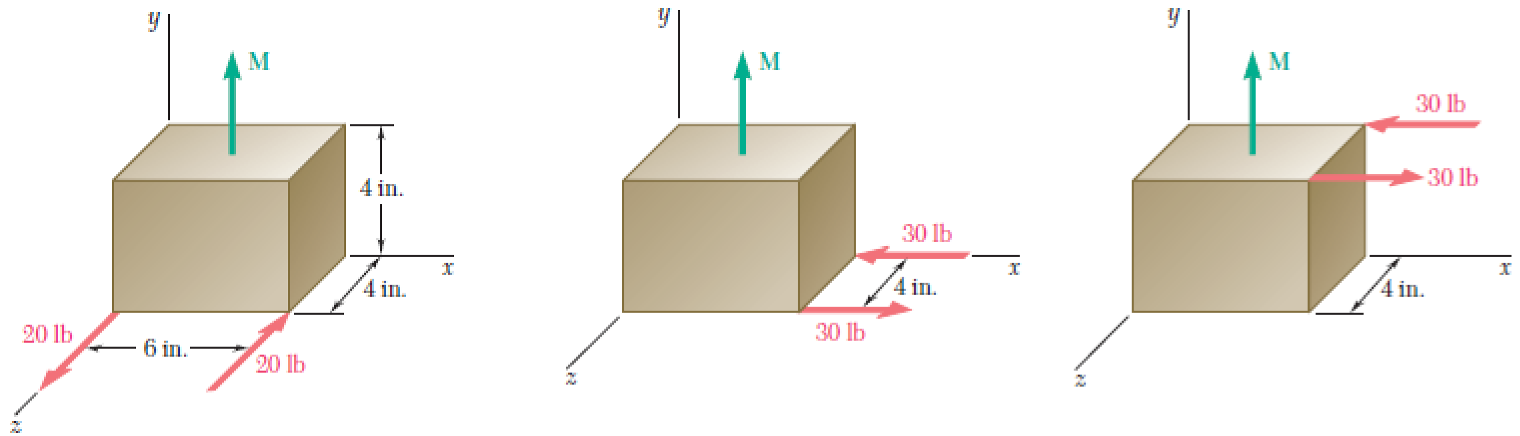


Principle of Transmissibility

The ***principle of transmissibility*** states that the conditions of equilibrium or motion of a rigid body will remain unchanged if a force ***F*** acting at a given point of the rigid body is replaced by a force ***F′*** of the same magnitude and same direction, but acting at a different point, *provided that the two forces have the same line of action*



EQUIVALENT COUPLES



(a) (b) (c)

- The property we have just established is very important for the correct understanding of the mechanics of rigid bodies. It indicates that when a couple acts on a rigid body, it does not matter where the two forces forming the couple act or what magnitude and direction they have. The only thing that counts is the *moment* of the couple (magnitude and direction). Couples with the same moment have the same effect on the rigid body.